Weak measurements and metrology

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Weak measurements

Discovered during exotic research on foundations of QM
Aharonov-Albert-Vaidman PRL 1988

USEFUL

- **Explore foundations of QM**

  Hardy paradox Aharonov et al. PLA 2002
  Observe wavefunction Lundeen et al. Nature 2011, Steinberg et al. Science 2011

- **Cavity QED experiments** Wiseman PRA 2002

- **Telecom optics** Brunner et al. PRL 2003

- **Superluminal propagation** Brunner et al. PRL 2004

- **Solid-state physics** Williams-Jordan PRL 2008

WM and Metrology

Precision measurements

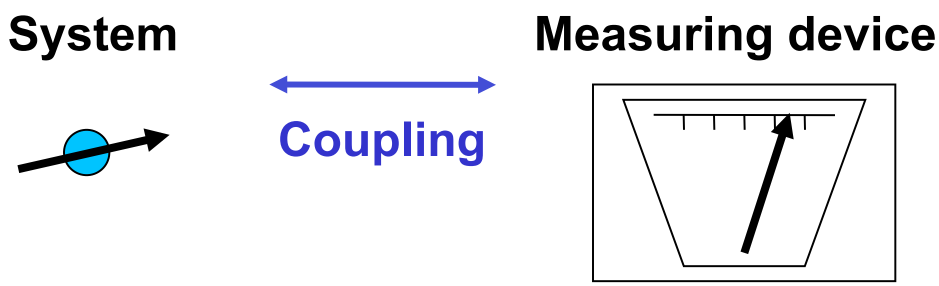
QUANTUM INSPIRED METROLOGY

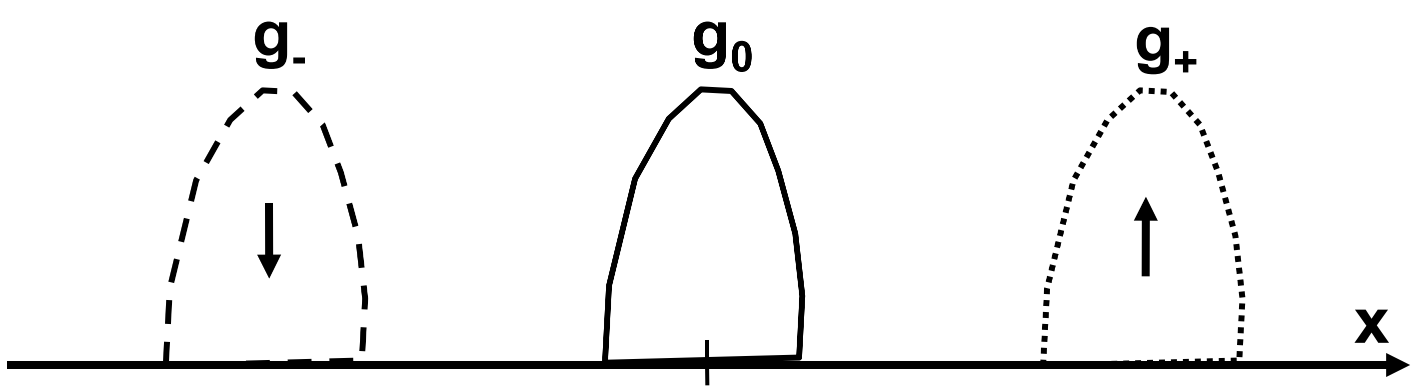
- **Spin Hall effect of light** (Hosten & Kwiat, Science 2008)
- **Small beam deflections ~15fm** (Dixon, Starling, Jordan, Howell PRL 2009)

HERE : SMALL PHASE SHIFTS, MAGNETOMETRY

What is a weak measurement?

Standard picture of Q measurement (von Neumann)

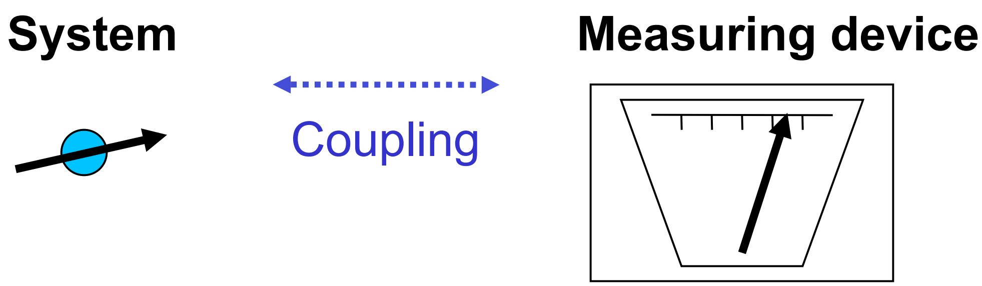


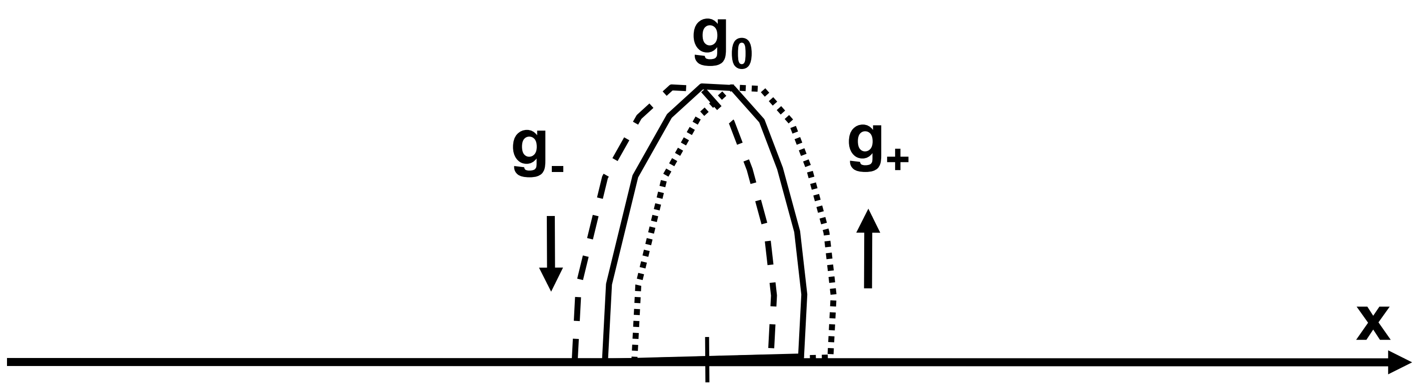


$$|\psi\rangle_S|g_0\rangle_P = (\alpha|\uparrow\rangle_S + \beta|\downarrow\rangle_S)\,|g_0\rangle_P \longrightarrow \alpha|\uparrow\rangle_S|g_+\rangle_P + \beta|\downarrow\rangle_S|g_-\rangle_P$$

Pointer read-out → Collapse of system / Full information

What is a weak measurement?



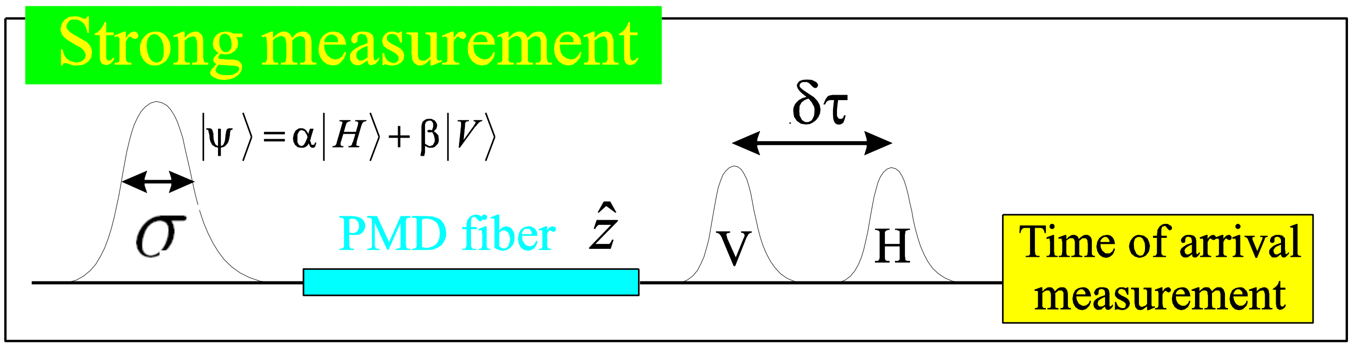


$$|\psi\rangle_S|g_0\rangle_P \rightarrow \alpha|\uparrow\rangle_S|g_+\rangle_P + \beta|\downarrow\rangle_S|g_-\rangle_P \simeq |\psi\rangle_S|g_0\rangle_P$$

Pointer read-out → no disturbance / little information

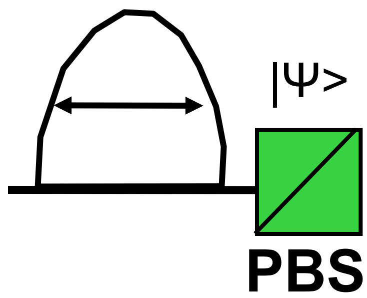
Example - Birefringence

Coupling between polarization and temporal mode



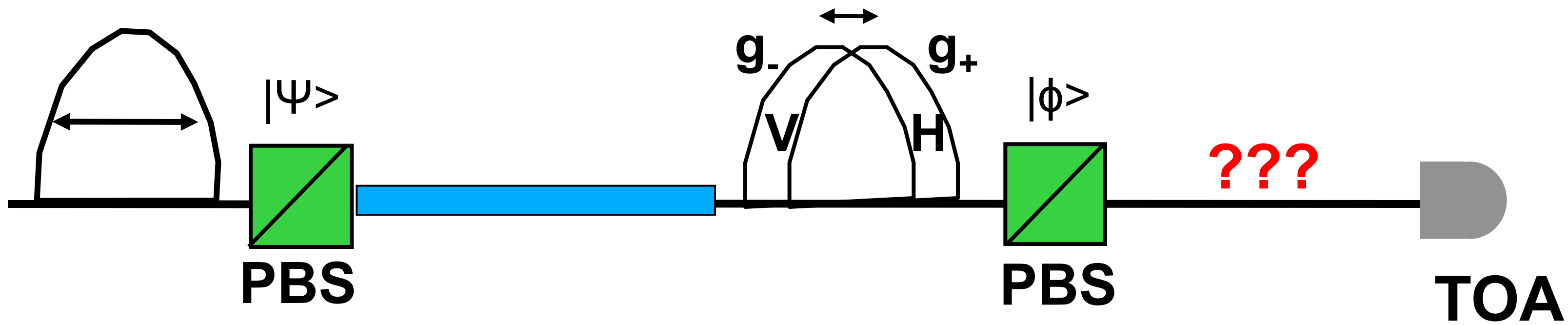
Brunner, Acin, Collins, Gisin, Scarani, PRL 2003

Post-selection - Weak value



1. Pre-selection $|\psi\rangle = \alpha|H\rangle + \beta|V\rangle$

Post-selection - Weak value

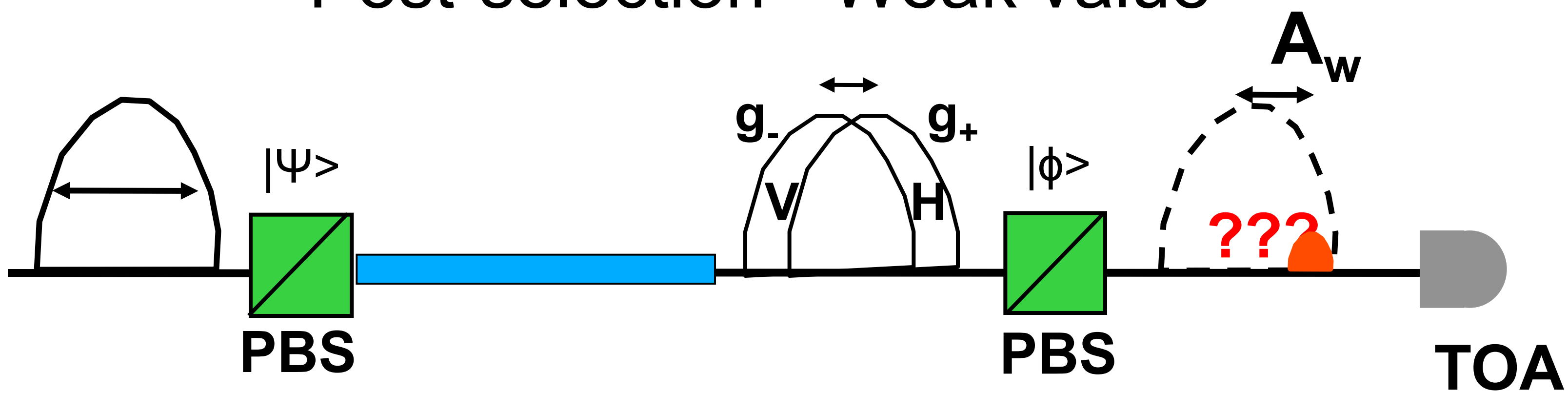


1. Pre-selection $|\psi\rangle = \alpha|H\rangle + \beta|V\rangle$

2. Weak meas. $U = e^{-i\tau\omega\sigma_z}$ Shift operator

3. Post-selection $|\phi\rangle = \mu|H\rangle + \nu|V\rangle$

Post-selection - Weak value



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Output state

$$|g_{out}\rangle = \langle\phi|e^{-i\tau\omega\sigma_z}|g(t)\rangle|\psi\rangle$$
$$\simeq \langle\phi|\psi\rangle e^{-i\tau A_w\omega}|g(t)\rangle$$

Amplification of pointer shift

$$A_w = \frac{\langle\phi|\sigma_z|\psi\rangle}{\langle\phi|\psi\rangle}$$

Weak value

Fastlight & Neutrinos

Desktop experiment

Brunner, Scarani, Legre, Wegmuller, Gisin, PRL 2004

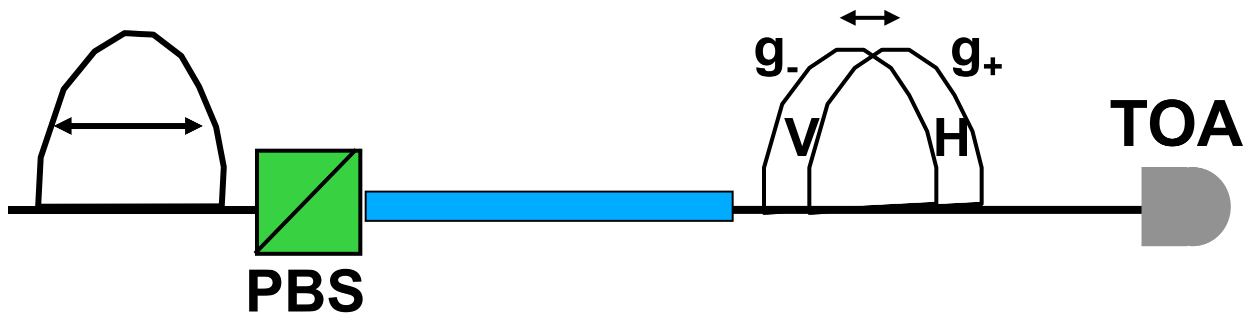
NEUTRINOS OSCILLATIONS ≈ BIREFRINGENCE

SUPERLUMINAL NEUTRINOS AT OPERA ? **PROBABLY NOT**

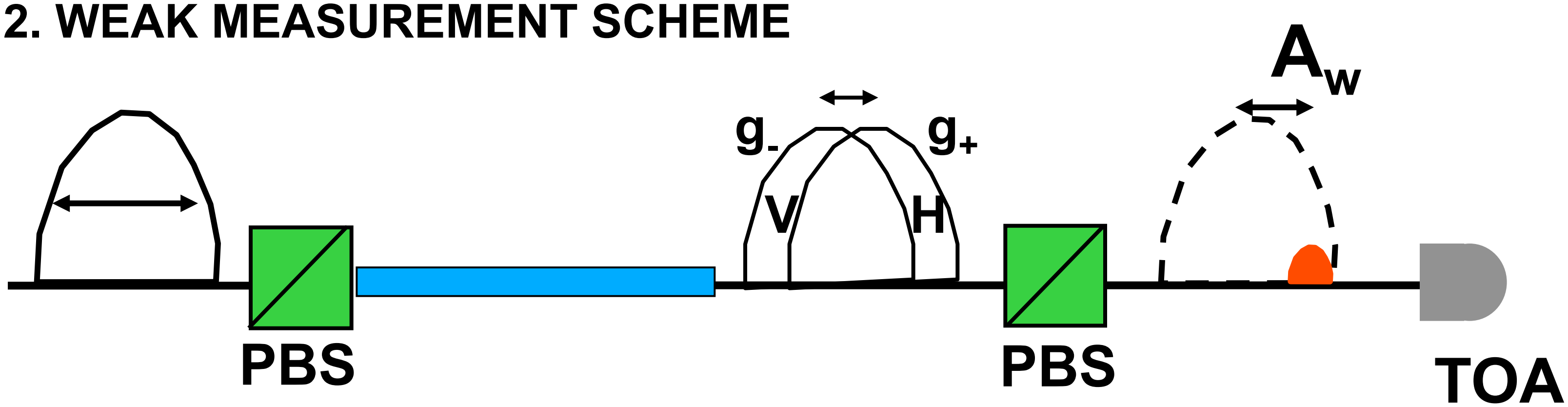
Berry, Brunner, Popescu, Shukla, J Phys A 2011

Metrology

1. DIRECT OBSERVATION



2. WEAK MEASUREMENT SCHEME



WM scheme

<u>Advantage:</u> Signal is amplified by the weak value

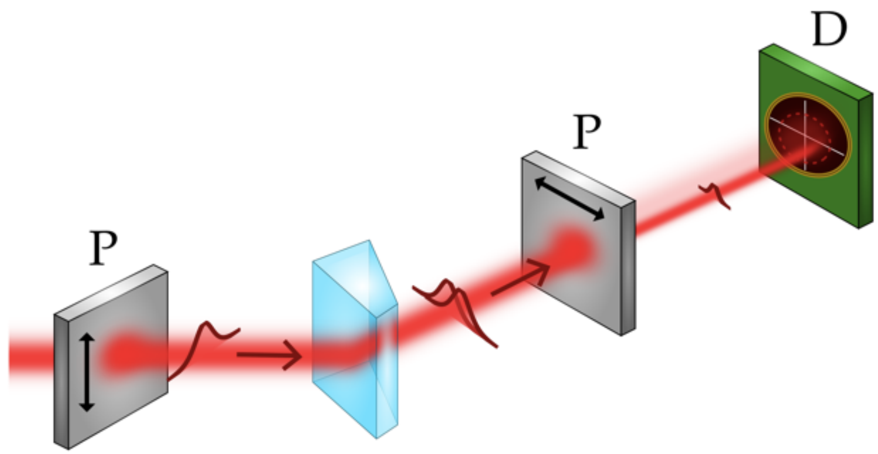
Amplification $A_w = \dfrac{\langle \phi | A | \psi \rangle}{\langle \phi | \psi \rangle}$

<u>Price to pay:</u> intensity of the signal is reduced

Prob. success $p \simeq \dfrac{1}{A_w^2}$

Useful for transverse effects

1. Spin Hall effect of light (Hosten-Kwiat 2008)
Effect ~ Å



2. Small beam displacement (Dixon et al. PRL 2009)
Effect ~15fm

What about longitudinal effects?

Measuring small phase shifts

Small phase shifts stable in time

<u>Limiting factor:</u> alignment errors (not photon statistics)

Compare 3 techniques

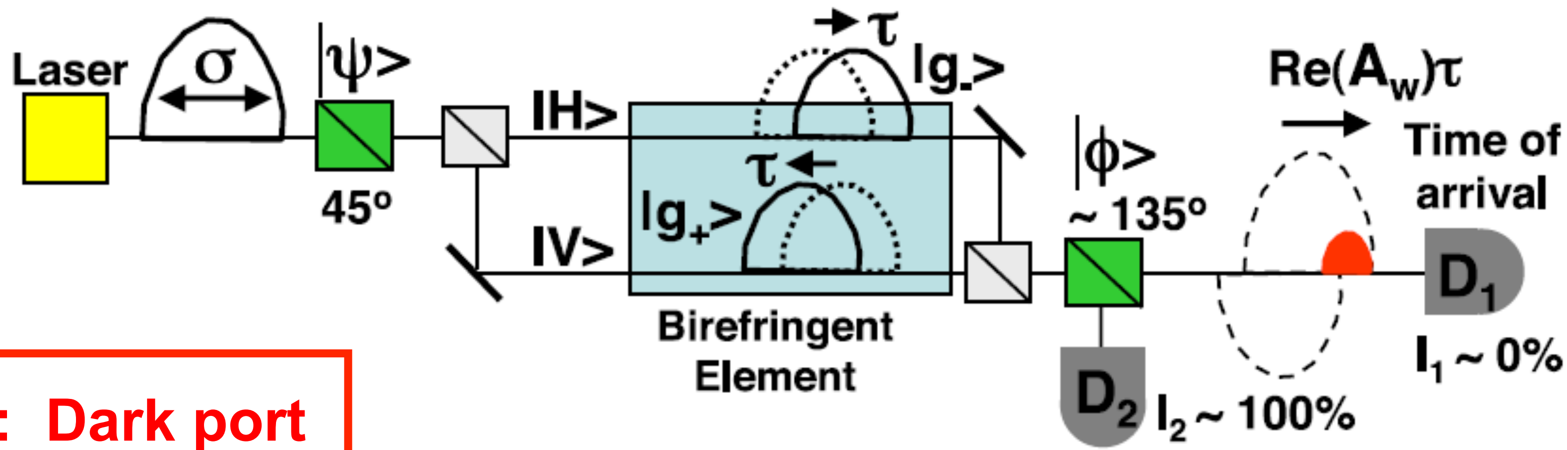
a) **Weak meas. Real weak value**

b) **Weak meas. Imaginary weak value**

c) **Standard interferometry**

Different operating point of IF and detection

a) Weak Meas. Real weak value



IF: Dark port
Det: Time

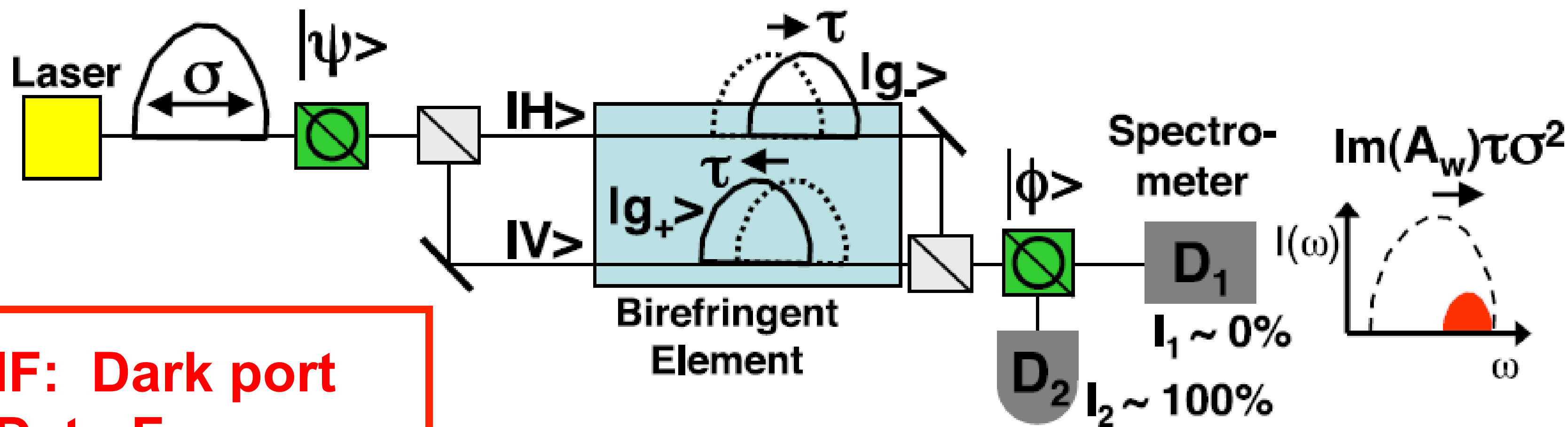
Time resolution of detector

Signal: $\tau A_w \simeq \tau/\sqrt{p} > \Delta t \longrightarrow \tau > \Delta t\sqrt{p}$

Angle misalignement ϵ: $P_{error} \simeq \epsilon^2 \longrightarrow p > \epsilon^2$

Resolution limit: $\boxed{\tau > \epsilon\Delta t}$

b) Weak Meas. Imaginary weak value



IF: Dark port
Det: Frequency

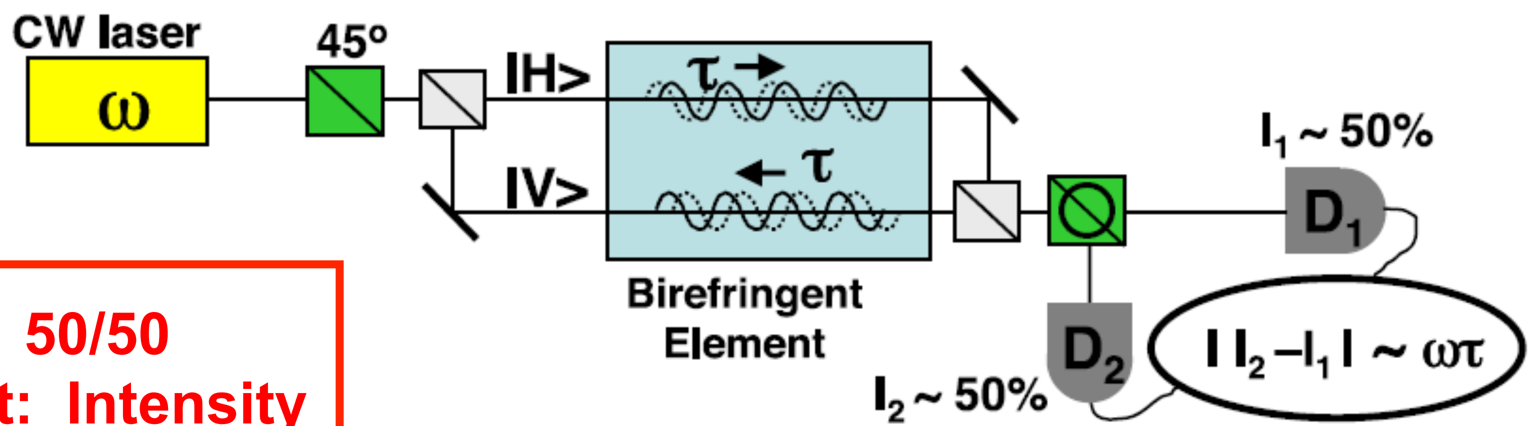
Spectral resolution

Signal: $\delta\omega \simeq \tau/(\sigma^2\sqrt{p}) \longrightarrow \delta\omega > \Delta\omega$

Angle misalignement ϵ: $P_{error} \simeq \epsilon^2 \longrightarrow p > \epsilon^2$

Resolution limit: $\boxed{\tau > \epsilon\sigma^2\Delta\omega}$

c) Standard Interferometry



IF: 50/50
Det: Intensity

Signal: $|I_1 - I_2| \simeq 2N\omega\tau$

Angle misalignement ϵ **:** $P_{error} \simeq \epsilon \longrightarrow \omega\tau > \epsilon$

Resolution limit: $\boxed{\tau > \frac{\epsilon}{\omega}}$

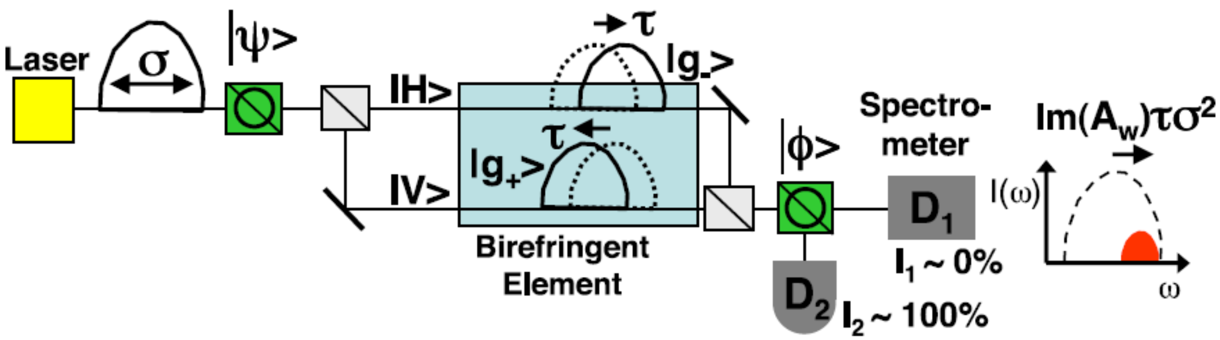
Comparison

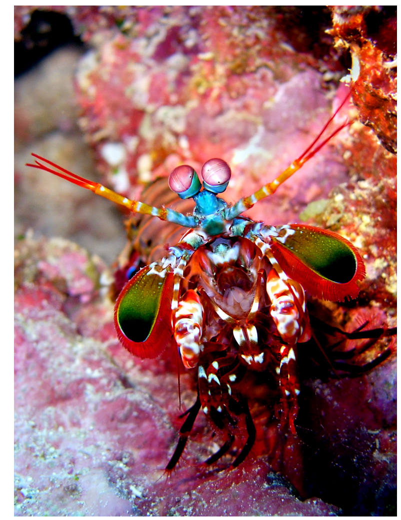
	Resolution limit	**In practice**
a) Real weak value	$\tau > \epsilon \Delta t$	$\boxed{\Delta t > 10 \text{ ps}}$ Gol'tsman et al. (2005)
c) Interferometry	$\tau > \frac{\epsilon}{\omega}$	$\boxed{1/\omega \sim 1 \text{ fs}}$ Optical frequencies
b) Imag. weak value	$\tau > \epsilon \sigma^2 \Delta \omega / 2$	$\boxed{\sigma^2 \Delta \omega = 0.5 \text{ as}}$

N. Brunner & C. Simon, PRL 2010

Perspectives

PRACTICAL IMPLEMENTATION ?





Roberts et al. Nature Phot 2009

OTHER METROLOGY APPLICATIONS ?
NASA ? LIGO ?

More

Weak measurements amplification ⟷ **Noiseless amplification of coherent states**

Ralph-Lund 2009
Exp : Pryde, Bellini, Leuchs

- Based on post-selection
- Heralded
- Achieve something that would be impossible deterministically

Brunner, Polzik, Simon PRA 2011

Heralded amplification of rotations in spin ensembles

→ **MAGNETOMETRY**

Thank you